Section 3.10

The Inverse Trig Functions

Recall the inverse trigonometric functions output the angle where the given trig function takes the desired value. For example, $\arcsin x = \theta$ is the angle θ such that $\sin\theta = x$.

Take a look at the three main trig functions to determine the domain and range of their respective inverses.

Practice 3.10.1. Compute

1. $\arcsin\left(\frac{\sqrt{3}}{2}\right)$

2. $\arccos\left(\frac{-\sqrt{3}}{2}\right)$

3. $\arccos(\cos(3\pi))$

In general we can use implicit differentiation to find the derivative of inverse functions:

Theorem 3.10.2. Let f be differentiable and have an inverse on an interval I. If a is a point such that $f'(a) \neq 0$, then f^{-1} is differentiable at $x = f(a)$ and

$$\frac{d}{dx}\big(f^{-1}(x)\big)\Big|_{x=f(a)} = \frac{1}{f'(a)}.$$

If f^{-1} is differentiable on the range of f, then

$$\frac{d}{dx}\big(f^{-1}(x)\big) = \frac{1}{f'(f^{-1}(x))}.$$

While we can use the above to help find the derivatives of inverse trig functions, we still will want to simplify the formulas to some extend.

Use implicit differentiation to find the derivative of $y = \arcsin x$.

How can we quickly find $\dfrac{d}{dx}\arccos x$?

Use implicit differentiation to find the derivative of $y = \arctan x$.

Use implicit differentiation to find the derivative of $y = \text{arcsec } x$.

Theorem 3.10.3 (Derivatives of Inverse Trig Functions)**.**

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\arccos x = \frac{-1}{\sqrt{1-x^2}}, \text{ for } -1 < x < 1$$

$$\frac{d}{dx}\arctan x = \frac{1}{x^2+1} \qquad \frac{d}{dx}\text{arccot } x = \frac{-1}{x^2+1}, \text{ for } -\infty < x < \infty$$

$$\frac{d}{dx}\text{arcsec } x = \frac{1}{|x|\sqrt{x^2-1}} \qquad \frac{d}{dx}\text{arccsc } x = \frac{-1}{|x|\sqrt{x^2-1}}, \text{ for } |x| > 1$$

Example 3.10.4.

1. Evaluate $f'(2\sqrt{3})$ when $f(x) = x\arctan(x/2)$.

2. Differentiate $y = \arctan 10x$

3. Differentiate $f(w) = \cos(\arcsin 2w)$

4. A small plane flies horizontally on a line 400 meters directly above an observer. Let θ be the angle of elevation of the plane. What is the rate of change of the angle of elevation $d\theta/dx$ when the plane is $x = 500$ m past the observer?